

feature of the book is a brief discussion of almost-periodic signals in the sense of H. Bohr [1], who is also known for work on Dirichlet series. Special attention is given to *finite-series* representations with *incommensurable* periods, although most of the results hold for infinite-series representations, too. Further, Parseval's theorem for almost-periodic functions is mentioned. Probability theory is considered in order to discuss the engineering uses of correlation functions and the notion of information. Finally, some basic nonlinear processes are discussed including such subjects as modulation and detection. A discussion or mention of Wiener's idea [2] of using shot noise as a standard signal for probing nonlinear systems would have added to the value of the monograph. Finally, if the author had treated Thévenin's theorem or Norton's theorem, the analysis of various interesting systems with a single nonlinear element would have been amenable.

The book should be especially useful for supplementary reading by undergraduates in electric science.

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1. H. BOHR, *Almost Periodic Functions*, Chelsea, New York, 1947.
2. N. WIENER, *Nonlinear Problems in Random Theory*, Technology Press and Wiley, New York, 1958.

73[P, X].—PATRICK BILLINGSLEY, *Ergodic Theory and Information*, John Wiley and Sons, Inc., New York, 1965, xiii + 193 pp., 23 cm. Price \$8.50.

This beautiful little book, which grew out of a series of lectures given by the author in 1963, centers about the Kolmogorov-Sinai theory of entropy of measure-preserving transformations.

The book begins with a discussion of the ergodic theorem, whose significance is illustrated by a well-chosen battery of examples drawn from analysis and from probability theory; the notion of ergodicity is explained; criteria for ergodicity are derived, and applied to various of the examples considered. A succinct proof of the ergodic theorem by the method of Riesz follows. Various more subtle examples of measure-preserving transformations are then discussed, including the standard measure-preserving transformation associated with the continued fraction expansion of real numbers lying in the unit interval, whose analysis, in a truly elegant section of 10 pages, yields numerous interesting measure-theoretic results concerning this expansion.

The second main, and the central, chapter of the book begins with a general discussion of the problem of isomorphism for measure-preserving transformations, following upon which the entropy invariant is defined, its properties established, and, after a technical section on separable measure spaces, its invariance proved. The nonisomorphism of certain measure-preserving shift transformations follows. An additional section discusses the spectral type of a measure-preserving transformation as another isomorphism invariant, and exhibits transformations of different entropies with identical spectral types. A final section in this chapter surveys some additional results and describes a number of open problems.

Next follows a chapter of a preparatory character which sets forth the basic

properties of the notion of conditional probability and expectation, and which concludes with a discussion of the convergence properties of conditional expectations. The results obtained are then used to derive the convergence properties of the entropy function, the Shannon-McMillan-Brieman theorem on convergence almost everywhere of the averaged logarithm of a conditional probability, and the equipartition property of ergodic shifts. A final section of the fourth chapter presents some interesting results of Eggleston and the author, relating the entropy of certain shift transformations to the Hausdorff measure of Cantor-like perfect sets on the real axis.

In the fifth and final chapter, the general theory is applied to the concrete problems of coding and information transmission, out of which the theory originally developed. A noiseless channel is modelled as a measurable transformation in a clear and convincing way; the proof that the entropy of a source is a lower bound for the capacity of a channel capable of transmitting the source without loss of information follows almost immediately. The converse result, i.e., the existence of a channel of capacity equal to the source entropy capable of transmitting the source, is reduced to a hypothesis concerning standard shifts. The results are then extended to noisy channels. Finally, the abstract existence-of-a-channel result is reformulated in concrete terms as a statement about the existence of block codes for the transmission of data from a given source, which is, of course, the form in which the result is of greatest interest to the practicing communications engineer.

The exposition is consistently crisp, succinct, unpretentious, and maximally clear. Professor Billingsley's work may be recommended not only to mathematicians wishing to become familiar with the interesting advances in ergodic theory on which he reports, and to communications scientists desiring insight into the theoretical foundations of information-rate theory, but also to potential authors, who will see in it an inspiring example of what the short survey monograph can be.

D. S.

74[P, X].—RAYMOND W. SOUTHWORTH & SAMUEL L. DELEEUW, *Digital Computation and Numerical Methods*, McGraw-Hill Book Co., Inc., New York, 1965, xiv + 508 pp., 23 cm. Price \$11.75.

According to the preface, "The aims of this course are (1) to introduce the student to numerical methods, as applied to the analysis and solution of engineering problems and (2) to develop enough facility in the programming of computers to allow him to solve problems on a digital computer." With this purpose in view the authors have divided the book into two main sections: a programming section comprising about a third of the book and a numerical analysis section comprising the remaining two thirds. Most chapters end with illustrative examples taken from various fields of engineering in addition to a large selection of problems for the student.

The section of the book devoted to programming covers flow charting and FORTRAN (FORTRAN IV) programming. The book also contains a chapter on machine language programming which appears to be too terse to eliminate the students' "black box" view of the computer. The chapter could have been deleted without damage to the book as a whole.

The scope of the numerical analysis section may be indicated by simply listing